2020 Asia and Pacific Mathematical Contest in Modeling

Problem A

Laser Marking Hatch Contour Generation

Laser is an important invention in the 20th century, and it is called “the sharpest knife”, “the most accurate ruler” and “the most unearthly light”. Laser has been increasingly applied to industrial processing, which can be employed in various processing operations such as marking, welding, drilling, cutting, heat treatment, and spray-coating. Remarkably featured by its fine directionality, laser can be converged into tiny sized spots so as to achieve the high centralization of energy; moreover, due to its sound monochromaticity, laser is easy to control.

Laser marking means using laser to mark LOGO, characters, symbols, images, etc. on the surface of products. It is a widely used processing method with its advantages of high processing efficiency, non-contact operation, no consumables, slight influences on product surface deformation, and firmness of marked content.

The hatch tool of laser marking machine can be used to hatch specified 2D-compound curve graph, and the setting of different hatch parameters have a great impact on the processing effects of different materials. The direction parallel hatch and the contour parallel hatch are two basic ways of hatching. The direction parallel hatch, also known as “zigzag” hatch, has paths being moved along line segments which are parallel to an initially selected reference direction. Based on this strategy a connected paths is obtained by linking these parallel segments so that they are either all traversed from right to left (or left to right) or, alternately from left to right and from right to left. Whereas the contour parallel hatch uses offset segments base the boundary curves as smooth hatch path that similar to the boundary curve. Thus, the contour parallel hatch be generated in a spiral-like fashion along curves that are at constant distances from the curve boundary. Which type of hatch is applied in practice highly depending on mark materials and the process effects on the particular machining task to be performed.

The hatching entities must be closed curvilinear polygons, and multiple mutually-nested contour objects may be filled by group-hatching. For the hatching process of figures, the existing boundary contours shall be offseted inward or outward firstly according to edge distance, before getting subject to the zigzag parallel or contour parallel hatch. The hatch curves shall be even and regular, and be kept basically parallel to each other. Neither having omissions of filling in an area, nor repeated filling of the area is allowed. The hatching by laser marking

shall generate hatched contours online in a real-time way. To meet the requirement of high-efficiency laser marking, **the hatch curves should stay parallel to the figures' boundary lines, distribute evenly to the greatest extent, and be generated automatically and quickly**. Efficiency is an important indicator for the generation of hatched figures.

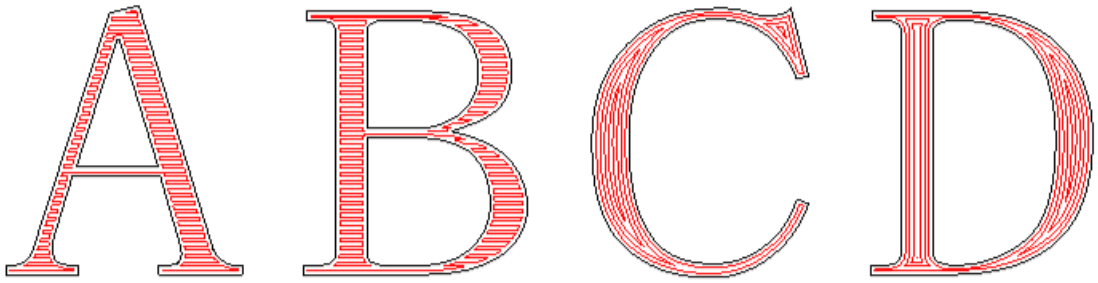

Figure 1. Graphical representation of zigzag parallel and contour parallel hatch

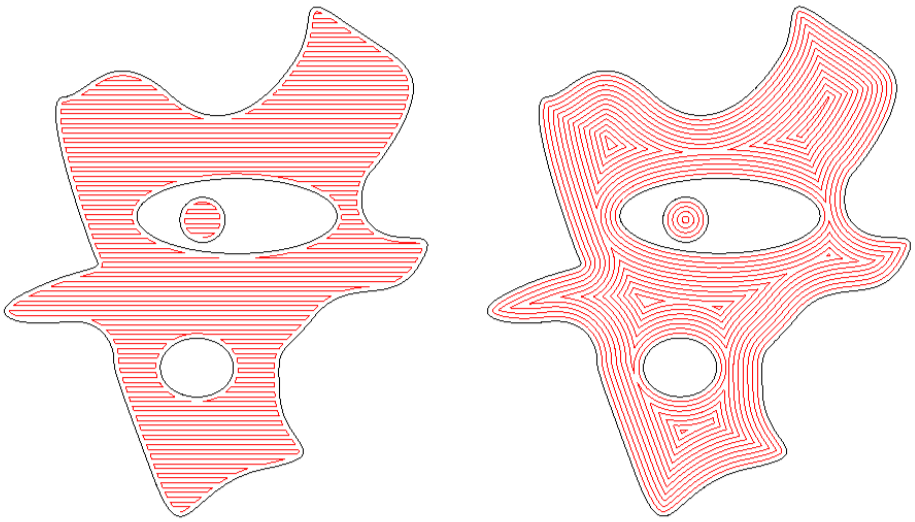

Figure 2 zigzag parallel and contour parallel hatch of multi-layer curves

(From outside to inside of nested areas, only odd layers are hatched.)

Please research the characteristics of the figures subject using zigzag parallel and contour parallel hatch, establish the mathematical model of hatching, design the algorithm, discuss the efficiency of algorithm, and answer the following questions:

1. Realize the zigzag parallel and contour parallel hatch of the single-layer contour pattern in Attachment 1; and here, only the hatch in horizontal direction (0° degree) is considered for zigzag parallel hatch. Please realize the hatching under two groups of input parameters according to the coordinate points data of the figure in Attachment 1 (unit: mm):

(1) Internal contraction of boundary distance 1mm, hatch line spacing 1mm;

(2) Internal contraction of boundary distance 0.1mm, hatch line spacing 0.1mm.

Under the two groups of parameters, calculate the total length of hatching lines of the hatched curves subject of zigzag parallel and contour parallel hatch, and count the number of horizontal lines of zigzag parallel hatch, and the number of circles of contour parallel hatch. Count the average elapsed time (unit: ms) based on the multiple runs of hatching program, and calculate the ratio of elapsed time for program running under conditions of parameter groups (2) and (1).

2. Realize zigzag parallel and contour parallel hatch of the mutually nested multi-layer contour patterns in Attachment 2; and here, only the hatching in horizontal direction (0° degree) is considered for zigzag parallel hatch. Please realize the hatching under two groups of input parameters according to the coordinate points data of the figure in Attachment 2 (unit: mm):

(1) Internal contraction of boundary distance 1mm, hatch line spacing 1mm;

(2) Internal contraction of boundary distance 0.1mm, hatch line spacing 0.1mm.

Under the two groups of parameters, calculate the total length of hatching lines of the hatched curves subject to zigzag parallel and contour parallel hatch , and count the number of horizontal lines of zigzag parallel hatch, and the number of circles of contour parallel hatch. Count the average elapsed time (unit: ms) based on the multiple runs of hatching program, and calculate the ratio of elapsed time for program running under conditions of parameter groups (2) and (1).

3. The diameter of light spots is very small when generated from laser marking machine. The hatching line spacing of laser marking is generally 0.01mm-0.1mm. Laser marking generates a large number of contour lines data, and the contour lines of any shape have extremely high requirements on the running efficiency of programs. Please check the elapsed time of your hatching algorithm and analyze its performance to provide a strategy or direction for optimizing the performance and efficiency of the hatching algorithm, so that it can meet the efficiency requirement of actual industrial applications.